Effective Actions from Loop Quantum Cosmology: Correspondence with Higher Curvature Gravity

Ghanashyam Date[1,*] and Sandipan Sengupta[1,†]

[1]The Institute of Mathematical Sciences

CIT Campus, Chennai-600 113, INDIA.

Abstract

Quantum corrections of certain types and relevant in certain regimes can be summarised in terms of an effective action calculable, in principle, from the underlying theory. The demands of symmetries, local form of terms and dimensional considerations limit the form of the effective action to a great extent leaving only the numerical coefficients to distinguish different underlying theories. The effective action can be restricted to particular symmetry sectors to obtain the corresponding, *reduced effective action.* Alternatively, one can also quantize a classically (symmetry) reduced theory and obtain the corresponding effective action. These two effective actions can be compared. As an example, we compare the effective action(s) known in isotropic loop quantum cosmology with the Lovelock actions, as well as with more general actions, specialized to homogeneous isotropic space-times and find that the $\bar{\mu}$-scheme is singled out.

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*[*]Electronic address: shyam@imsc.res.in

[†]Electronic address: sandi@imsc.res.in

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I.   INTRODUCTION

It is quite common to incorporate various types of quantum corrections in an *effective action* which contains the classical action one begins with. The degree of generality chosen for the form of an effective action, reflects the context of its proposal/computation and usually accounts for a class of quantum corrections. For example, the effective action formally defined from the path integral is in general expected to be non-local. However, in perturbation theory it is typically obtained as an infinite power series each term of which is *local* in the basic fields and their derivatives. It incorporates the perturbative quantum corrections. Such effective actions can be constructed starting from any classical action, in particular for both a "full theory" and its "reduced versions" corresponding to some chosen (classical) sectors thereof. The same reduction procedure can be carried out for the full theory effective action, possibly with further approximations. Thus we have two effective actions and a comparison is conceivable. Such comparisons could shed some light on quantize-after-reduction and reduce-after-quantization approaches. However, if one has only effective actions for different classical sectors of a theory, then the demand that these be obtainable from a common full theory effective action could be used to constrain some of the quantization ambiguities of the reduced models.

We attempt such an exercise in the context of the effective actions available in the loop quantization of the homogeneous and isotropic sector of Einstein's theory. Assuming the usual higher derivative effective actions for the full theory, it follows that of the various quantization schemes available in isotropic loop quantum cosmology (LQC), the so-called $\bar{\mu}$-scheme is the most natural one. The scope and limitations of such comparisons is also discussed.

The paper is organized as follows. In section II, we take the effective Hamiltonian from LQC and obtain a corresponding Lagrangian in a suitable form. These are written in a form which subsumes the old μ_0-scheme, the improved $\bar{\mu}$-scheme as well as the form coming from lattice refinement models. In section III, we consider the Lovelock action in arbitrary space-time dimensions, specialize them for the FRW form of metric and compare with the LQC effective actions. In the next section, we discuss the more general forms of actions, in four space-time dimensions, specialise to the FRW metric and discuss methods for comparing with the LQC effective action. The final section contains a summary and some remarks.

II. EFFECTIVE ACTION FROM LQC

Let us consider the simplest of the homogeneous models, namely the isotropic model suitably coupled to scalar matter. The fundamental discreteness of the LQC implies two distinct types of corrections: (a) those arising from replacement of connections by holonomies and (b) those arising due to the unusual definition of the inverse volume operator forced by the discreteness. The latter ones typically show up in the matter sector and are absent in the gravitational part for the spatially flat, isotropic models. The modifications implied by these are small deviations in the classical regime: $\ell_\mathrm{P} \frac{\dot{a}}{a} \ll 1$. In this regime, the corrections are summarised in an *effective Hamiltonian* which has been obtained from the quantum Hamiltonian constraint using a *leading order* WKB approximation [1] or via expectation values in the (kinematical) coherent states [2]. In the gravitational sector it takes the form,

$$H_\mathrm{grav} = -\frac{3}{\kappa}\sqrt{p}\left[\frac{\sin^2\left(\epsilon(\alpha,p)K\right)}{\epsilon^2(\alpha,p)}\right] \quad, \quad \{K,p\} = \frac{\kappa}{3} \ , \ \kappa := 8\pi G \ ,$$

$$\epsilon(\alpha,p) := \mu(\alpha,p)\gamma \ , \quad \mu(\alpha,p) := \mu_\alpha\left(\frac{\sqrt{\gamma}\ell_\mathrm{P}}{\sqrt{p}}\right)^{-2\alpha} . \tag{1}$$

The K is related to the usual 'connection variable' c by $K := c\gamma^{-1}$. The parameter α is used to denote various quantization schemes. μ_α is a constant, γ is the Barbero-Immirzi parameter and ℓ_P is the Planck length (its precise definition does not matter for our purpose).

The older quantization scheme [3] is obtained for $\alpha = 0$ with $\mu(0,p) := \mu_0$, a fixed ambiguity parameter. The improved quantization of [4] is obtained for $\alpha = -\frac{1}{2}$ with $\mu_{-1/2} := \Delta$ while [5] permits all values of α, $-1/2 \le \alpha \le 0$. Note that the classical Hamiltonian is recovered in the limit $\epsilon(\alpha,p) \to 0$.

We will first make a simple "canonical" transformation so that the scale factor is the configuration space variable and then do an inverse Legendre transformation to get the corresponding Lagrangian which is a function of only the scale factor and its time derivative. This is the form that we will compare with a general form of an effective action specialized to the FRW metric.

We begin by introducing the identification $a := \xi\sqrt{p}$ and its conjugate variable $p_a(K,p)$ to be chosen such that $\{a,p_a\} = \frac{\kappa}{3}$. This leads to a choice, $p_a(K,p) := -\frac{2\sqrt{p}}{\xi}K$. Substituting $\sqrt{p} = a\xi^{-1}$, $K = -\frac{\xi^2}{2}\frac{p_a}{a}$ in the Hamiltonian leads to,

$$H_\mathrm{grav}(a,p_a) = -\frac{3}{\kappa}\frac{a}{\xi}\frac{1}{\epsilon^2(\alpha,a)}\sin^2\left(\frac{\epsilon(\alpha,a)\xi^2 p_a}{2a}\right) \quad, \quad \{a,p_a\} = \frac{\kappa}{3} . \tag{2}$$

For the synchronous time (lapse equal to 1), one gets,

$$\dot{a} \;=\; \{a, H_{\text{grav}}(a, p_a)\} \;=\; -\frac{\xi}{2\epsilon(\alpha, a)}\sin\left(\frac{\epsilon(\alpha, a)\xi^2 p_a}{a}\right) \;. \tag{3}$$

The Lagrangian is obtained by inverse Legendre transformation,

$$L(a, \dot{a}) \;:=\; \frac{3}{\kappa}\dot{a}p_a(a, \dot{a}) - H_{\text{grav}}(a, p_a(a, \dot{a})) \;, \tag{4}$$

where $p_a(a, \dot{a})$ is to be obtained by inverting (3). This is easily done and gives,

$$\sin^2\left(\frac{\epsilon(\alpha, a)\xi^2 p_a}{2a}\right) \;=\; \frac{1 - \sqrt{1 - \frac{4\epsilon^2(\alpha, a)\dot{a}^2}{\xi^2}}}{2} \tag{5}$$

$$H_{\text{grav}}(a, \dot{a}) \;=\; -\frac{3}{2\kappa}\frac{a}{\xi}\frac{1}{\epsilon^2(\alpha, a)}\left[1 - \sqrt{1 - x^2}\right] \;,\quad x \;:=\; \frac{2\epsilon(\alpha, a)\dot{a}}{\xi} \tag{6}$$

$$L(a, \dot{a}) \;=\; \left[\frac{3}{\kappa}\right]\dot{a}\left\{-\frac{a}{\epsilon(\alpha, a)\xi^2}\sin^{-1}(x)\right\} - H_{\text{grav}}(a, \dot{a})$$

$$\;=\; -\left[\frac{3}{2\kappa}\right]\left[\frac{a}{\xi\epsilon^2(\alpha, a)}\right]\left[x \sin^{-1}x - 1 + \sqrt{1 - x^2}\right] \;. \tag{7}$$

The third bracket can be expressed as a power series in x as,

$$x \sin^{-1}x - 1 + \sqrt{1 - x^2} \;=\; \sum_{n=1}^{\infty}\frac{x^{2n}}{n!2^n}\frac{(2n - 3)!!}{(2n - 1)} \;, \tag{8}$$

where $n!! := 1 \cdot 3 \cdot 5 \cdots [n/2]$ and equals 1 for $n = 0$.

Observe that the $\epsilon(\alpha, a)^{-2}$ factor cancels, leading to the Lagrangian,

$$L(a, \dot{a}) \;=\; -\left[\frac{3}{\kappa}\frac{a\dot{a}^2}{\xi^3}\right]\left[1 + \sum_{n=1}^{\infty}\frac{x^{2n}}{(n+1)!2^n}\frac{(2n-1)!!}{(2n+1)}\right] \;, \tag{9}$$

$$x \;=\; \begin{cases} 2\mu_0\gamma\xi^{-1}\dot{a} & (\mu_0 - \text{quantization}) \\[2mm] 2\Delta\gamma^{3/2}\frac{\ell_{\text{P}}}{a}\dot{a} & (\bar{\mu} - \text{quantization}) \\[2mm] 2\mu_\alpha\gamma\left(\frac{\xi\sqrt{\gamma}\ell_{\text{P}}}{a}\right)^{-2\alpha}\xi^{-1}\dot{a} & (\text{lattice refinement}) \end{cases} \;.$$

For comparison with the classical theory, we recall that for the FRW spatially flat metric, one has

$$ds^2 \;:=\; dt^2 - a^2(t)\{dx^2 + dy^2 + dz^2\}$$

$$R \;=\; -6\frac{\ddot{a}}{a} - 6\frac{\dot{a}^2}{a^2} \;,$$

$$S \;:=\; -\frac{1}{16\pi G}\int dt \int_{\text{cell}} d^3x \sqrt{|\det g|}R \;=\; -\frac{3V_0}{\kappa}\int dt\, a\dot{a}^2 \;. \tag{10}$$

Here V_0 is the comoving volume of a fiducial cell necessary in an action formulation. Comparison of the first term of (9) and (10) suggests the identification $\xi = V_0^{-1/3}$.

Notice that only for $\alpha = -1/2$, does the dependence on the fiducial cell (through ξ) disappear and the Lagrangian becomes a power series in $\ell_{\rm P} \frac{\dot{a}}{a}$. In all cases, the degrees of freedom remain exactly the same and only the specification of the dynamics deviates from the Einsteinian one.

Now the question which is raised many times is whether the loopy quantum corrections that have been summarised in the LQC effective actions above, are "analogous" to the higher derivative terms expected in an effective action for the full theory. One way to explore this question is to look for an effective action for full theory, restrict it to the homogeneous and isotropic sector and compare with the LQC effective action(s). If an action for the full theory continues to lead to a second order (in time) field equation, then the degrees of freedom remain the same as those implied by the Einstein-Hilbert action and on restriction to the FRW sector, the same feature will continue to hold. Such actions are indeed available and are known as the Lovelock actions. We discuss these and their reduction, in the next section.

III. LOVELOCK ACTIONS

There is a special class of actions involving homogeneous polynomials in the Riemann tensor, Ricci tensor and Ricci scalar which have the property that the corresponding equations of motion are second order in time. These are the Lovelock actions [6]. If these are specialised to the FRW metrics, then they become a function of only $a, \dot{a}$ (up to total time derivatives). In the following, the space-time is taken to be D dimensional.

The pure Lovelock terms, L_n, are $2n$-th order homogeneous polynomials in the R_{abcd}, R_{ab} and R, with coefficients chosen so as to remove higher derivative terms. Explicitly [1],

$$L_n^D = \frac{1}{2^n} \, \delta_{b_1 \ldots b_{2n}}^{a_1 \ldots a_{2n}} \, R_{a_1 a_2}{}^{b_1 b_2} \cdots R_{a_{2n-1} a_{2n}}{}^{b_{2n-1} b_{2n}}, \tag{11}$$

where $\delta_{b_1 \ldots b_{2n}}^{a_1 \ldots a_{2n}}$ is the Kronecker symbol of order $2n$ (totally antisymmetric in both sets of indices) and R_{abcd} is the D-dimensional Riemann tensor. One may notice that $L_0^D = 1$

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[1] Here we follow the same notation as in [7].

corresponds to the cosmological constant term while the $L_1^D = R$ is the familiar Einstein Hilbert term. Next comes the Gauss-Bonnet term

$$L_2^D = R^2 - 4 R_{ab}R^{ab} + R_{abcd}R^{abcd}. \tag{12}$$

and so on. The L_n^D has dimensions of $(\text{length})^{-2n}$.

Due to the antisymmetrization, in D-dimensions, all Lovelock terms with $n > \frac{D}{2}$ vanish identically. For even D, the $L_{D/2}^D$ is a total derivative, and thus doesn't contribute to the equations of motion. For even D the $\sqrt{|g|}L_{D/2}^D$ is in fact the Euler density, a topological invariant for the manifold. The Lovelock action in D dimensions is a *linear combination* of the non-vanishing pure Lovelock terms. To facilitate comparison with the LQC, we drop the cosmological constant term.

Consider now the FRW metric in D space-time dimensions.

$$
\begin{aligned}
ds^2 &= dt^2 - a^2(t) \left[\frac{dr^2}{1 - kr^2} + r^2 d\chi_1^2 + r^2 sin^2\chi_1 d\chi_2^2 + \cdots \right] \\
&= dt^2 - a^2(t) \left[\frac{dr^2}{1 - kr^2} + r^2 d\Omega_{D-2}^2 \right] \quad \text{where,}
\end{aligned}
\tag{13}
$$

$a(t)$ is the scale factor, r is the radial coordinate while χ_i are the angular coordinates on the $(D-2)$ dimensional sphere. We consider the spatially flat case $(k = 0)$ throughout.

The non-zero components of the Riemann and the Ricci tensors are given by,

Riemann :

$$R^{tr}_{\ tr} = R^{t\chi_i}_{\ t\chi_i} = -\frac{\ddot{a}}{a} \quad , \quad R^{r\chi_i}_{\ r\chi_i} = -\frac{\dot{a}^2}{a^2} \quad , \quad i = 1, \cdots, D-2 . \tag{14}$$

$$R^{\chi_i\chi_{i+j}}_{\ \chi_i\chi_{i+j}} = -\frac{\dot{a}^2}{a^2} \quad , \quad i = 1, \cdots, D-2 , \quad j = 1, \cdots D-2-i .$$

Ricci :

$$R^r_{\ r} = -\frac{\ddot{a}}{a} - (D-2)\frac{\dot{a}^2}{a^2} \quad , \quad R^t_{\ t} = - (D-1)\frac{\ddot{a}}{a} \quad ,$$

$$R^{\chi_i}_{\ \chi_i} = R^r_{\ r} \quad , \quad i = 1, \cdots, D-2 . \tag{15}$$

The Ricci scalar is given by

$$R = - (D-1) \left[2\frac{\ddot{a}}{a} + (D-2)\frac{\dot{a}^2}{a^2} \right] \tag{16}$$

Using these expressions we obtain[2],

$$\sqrt{|g|}L_n^D = a^{D-1} \left[-\frac{(D-1)(D-2)\cdots(D-2n)}{(2n-1)} \right] \left(\frac{\dot{a}}{a} \right)^{2n} + \text{Total time derivative} \tag{17}$$

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[2] Explicit expressions of L_2, L_3 etc. in terms of the curvature invariants can be found in [8].

For a given D, the reduced Lagrangian is obtained as,

$$
\begin{aligned}
L_{\text{reduced}}^D &:= \sum_{n=1}^{[D/2]} \alpha_n \lambda^{2n} \int_{\text{cell}} \sqrt{|g|} L_n^D \\
&= \sum_{n=1}^{[D/2]} \xi^{1-D} \alpha_n \lambda^{2n} a^{D-1} \left[-\frac{(D-1)(D-2)\cdots(D-2n)}{(2n-1)} \right] \left(\frac{\dot{a}}{a} \right)^{2n} \\
&= -\left[\xi^{1-D} a^{D-1} \alpha_1 (D-1)(D-2) \left(\lambda \frac{\dot{a}}{a} \right)^2 \right] \times \\
&\qquad \left[1 + \sum_{n=2}^{[D/2]} \frac{\alpha_n}{\alpha_1} \left[\frac{(D-3)(D-4)\cdots(D-2n)}{(2n-1)} \right] \left(\lambda \frac{\dot{a}}{a} \right)^{2n-2} \right] \\
&= -\left[\xi^{1-D} a^{D-1} \alpha_1 (D-1)(D-2) \left(\lambda \frac{\dot{a}}{a} \right)^2 \right] \times \\
&\qquad \left[1 + \sum_{n=1}^{[D/2]-1} \frac{\alpha_{n+1}}{\alpha_1} \left[\frac{(D-3)(D-4)\cdots(D-2n-2)}{(2n+1)} \right] \left(\lambda \frac{\dot{a}}{a} \right)^{2n} \right] \quad (18)
\end{aligned}
$$

In the above λ is a constant with dimensions of length so that each of the term in the sum has the same dimension. The α_n/α_1 are arbitrary dimensionless constants and we take the cell to have the comoving volume given by ξ^{1-D} where ξ^{-1} is another length scale. Note that α_1 could be a dimensionful parameter.

Comparison of the second square brackets in (9) and (18) suggests the choices for λ and α_n/α_1, $n = 1, 2, \cdots, [\frac{D}{2}] - 1$:

$$
x \leftrightarrow \lambda \frac{\dot{a}}{a} \implies \lambda := 2\Delta \gamma^{3/2} \ell_{\mathrm{P}} \ ,
$$

$$
\frac{\alpha_{n+1}}{\alpha_1} \leftrightarrow \frac{2n+1}{(D-3)(D-4)\cdots(D-2n-2)} \cdot \frac{(2n-1)!!}{(n+1)! 2^n (2n+1)} \ . \quad (19)
$$

Note that λ being constant, selects the $\bar{\mu}$-scheme ($\alpha = -1/2$).

Matching the powers of a in the first square brackets however requires $D = 4$ and determines $\alpha_1 = (2\kappa \lambda^2)^{-1}$. But then the sum over n drops out. There does not seem to be a way to define any $D \to \infty$ limit such that (a) the finite sum can be extended to an infinite power series *and* (b) the first factors match.

Thus, although it is possible to get a reduced Lagrangian which depends only on $a, \dot{a}$, from a Lagrangian with higher powers of curvatures, this requires Lovelock Lagrangian in arbitrarily high space-time dimensions to generate the infinite power series in $\dot{a}/a$. In addition, the first factors do not match. This route for seeking an interpretation of the quantum corrections summarised in LQC effective action is not viable.

There is another way to obtain second order equation for the FRW sector from a general effective action which we discuss in the next section.

IV. GENERAL EFFECTIVE ACTION

In quantum field theory one constructs an effective action, formally, from the Feynman path integral [9]. In various approximations, one attempts to compute it. Weinberg [10] has given a general characterization of an effective action (*not to be requantized*) which is supposed to incorporate at least a class of quantum corrections. One chooses a set of fields, assumes certain invariances and also *locality* in the sense that the action is to be made up of terms each of which is an integral over positive integer powers of fields and their derivatives. For a quantized gravity [3], the field would be the metric tensor, its derivatives would be expressed in terms of the Riemann and Ricci curvatures and the invariance demanded is the general covariance [4]. Thus the general form is expected to be a power series in scalars constructed from the Riemann tensor, the Ricci tensor and the Ricci scalar. One can also put in the cosmological constant term. The general action will be an infinite series in these scalars whose *coefficients would depend on specific underlying quantum theories.* Within this class of effective actions, different proposals for a fundamental quantum field theory are distinguished only by these coefficients.

These coefficients are in general dimensionful. Since the Riemann tensor has length dimension of -2, different powers will have different dimensions while the action must be dimensionless. This fixes the dimensions of the coefficients. Observe that quantum gravity provides a natural length scale, namely the Planck length ℓ_P. So one can always use ℓ_P to convert the coefficients to dimensionless numbers which encode the specifics of the underlying quantum gravity theory.

Now the observation is that these coefficients are *independent of the field configurations* and one can hope to compare different theories by specializing to various physical contexts, such as the FRW metric, the metrics of diagonalised homogeneous models, spherically sym-

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[3] The idea that general relativity can be interpreted as an effective field theory by introducing higher order curvature invariants in the original action is discussed elaborately in ref.[11].

[4] We could also include covariant derivatives of the Riemann and Ricci tensors, but for our purposes, these will not be needed.

metric metric etc. In effect, one is carrying out (say) a symmetry reduction after quantization albeit only incorporating those features which are captured by the form taken for the general effective action. In contrast, the LQC effective actions derived above, incorporate a subset of corrections (the holonomy corrections in the gravitational sector) in a quantization of *a classically reduced theory.*

The effective action framework thus affords on the one hand a comparison of different fundamental theories and on the other hand a comparison of reduction after quantization and quantization after reduction approaches. These comparisons are of course limited due to inclusion of only a subset of corrections.

With these general remarks, let us consider the context of flat FRW models.

The locality assumption mentioned above is valid in a perturbative analysis (splitting the metric in a background and a fluctuation). In the context of a time dependent scale factor, perturbation would be appropriate only for a slow variation. Furthermore, quantum effects would be expected to be small when the background is 'almost classical' which in the context of flat, isotropic model, corresponds to "late time", $\ell_P \frac{\dot{a}}{a} \ll 1$.

As can be seen explicitly from the equations (14, 15, 16), the scalars constructed from polynomials in Riemann and Ricci tensors will be polynomials in $\frac{\ddot{a}}{a}, \frac{\dot{a}}{a}$. If we allowed derivatives of these tensors, then higher time derivatives would also be present. The general effective action would then be an infinite series in $H := \frac{\dot{a}}{a}$ and its time derivatives (apart from a^3 from the $\sqrt{g}$ factor). Note that the classical action part, modulo a total time derivatve, has no derivatives of H.

At this stage further approximations to the above action are conceivable. A general higher derivative action will have many more solutions than those of the leading order (classical) action. The perturbative nature of the quantum corrections should lead to small deviations from the classical solutions for self consistency. In particular, the space of classical solutions should remain the same (although the individual solutions will of course change). In effect, this requires the higher derivative terms to be thought of as being determined by the classical solutions, corrected order-by-order. The effective action is then again a polynomial in H, although not manifestly so. This is a correct procedure to interpret the higher derivative action obtained in a perturbative context [5]. Note that the same arguments also apply for

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[5] We thank an anonymous referee for drawing our attention to this point.

obtaining corrections to the classical solutions from the LQC effective action. However, the LQC effective action being a function of only H, one can keep this procedure implicitly understood.

Recall that the LQC Lagrangian of equation (9), is only the leading order of the WKB method and hence has only the classical degrees of freedom visible. The higher order corrections from the WKB, will introduce higher time derivatives as well which however are not yet available. In effect, one has implicitly *dropped the higher derivative corrections coming from LQC*. Had these terms been available in LQC effective action, one would apply the same considerations as given above.

At the present state of availability of quantum corrections from LQC, it seems more 'fair' to approximate the full theory effective action by explicitly dropping all terms containing derivatives of H, as has been implicitly done for the LQC effective action. Now both actions have same form and comparison of coefficients is possible.

To summarise, there are two ways to compare the two effective actions depending upon the form in which they are available. If both effective actions have higher derivatives, then in each one, the higher derivatives can be treated as being determined by solutions of the lower order equation of motion. In effect, one can compare the solutions connected to the *same* classical solution. Alternatively, if one action has no higher derivative terms, then the second one can be brought to the same form by dropping the higher derivative terms. Now the actions themselves can be compared directly.

Either of these is a possible method by which one can compare the LQC effective Lagrangian with an effective Lagrangian constructed for the full theory in any particular version of quantum gravity. It is also possible to carry out a similar comparison when effective Lagrangians become available for anisotropic LQC, spherically symmetric models etc. The full theory effective action will have the *same* coefficients in all these cases.

Here we note another point relevant for comparisons. Suppose an effective action is given as a series in curvature scalars with certain specific coefficients, $L \sim c_1 R + c_{2,0} R^2 + c_{2,1} R_{ab} R^{ab} + c_{2,2} R_{abcd} R^{abcd} + \cdots$ which is a function of $\dot{H}$ and H. In either of the methods described above, each of the curvature scalars will effectively be a monomial in H with the power determined by the dimensional consideration and the coefficient determined by actual computation. These are fixed coefficients independent of the quantum theory. For example, we could get $R^2 \approx k_{2,0} H^4$, $R_{ab} R^{ab} \approx k_{2,1} H^4$, $R_{abcd} R^{abcd} \approx k_{2,2} H^4$ and so on. The net result

will be a power series in H whose coefficients will be combinations of $c_{m,n}$ and $k_{m,n}$[6]. The H^4 coefficient for instance would be $(c_{2,0}k_{2,0} + c_{2,1}k_{2,1} + c_{2,2}k_{2,2})$. If we were to attempt inferring the theory dependent coefficients $c_{m,n}$ by a comparison, then the FRW sector can at best yield some constraints on the combinations and other (less symmetric) sectors will be needed.

V. SUMMARY AND CONCLUSIONS

In this work, we first obtained the effective Lagrangian(s) for the gravitational sector of isotropic LQC. The domain of validity of the effective Hamiltonian (and hence the Lagrangian) is the regime $\ell_{\mathrm{P}}|\dot{a}/a| \ll 1$. This is given as an infinite power series. There are two ways to obtain such series from an effective action for the full theory: (a) using actions whose degrees of freedom exactly match with the classical ones (same order of equations of motion), eg the Lovelock actions and (b) invoking a suitable approximation to restrict to the classical degrees of freedom as the dominant/relevant ones (directly by dropping higher derivative terms or indirectly by treating higher derivatives as being determined by lower order solutions). The former however requires considering arbitrarily high space-time dimensions and does not yield a form consistent with the LQC effective action. The latter, though it involves an approximation, is more general and consistent with the domain of validity of the LQC effective action as well as with the nature of quantum corrections implicit in the higher curvature action. This naturally restricts the LQC effective action to the $\bar{\mu}$-scheme. We would like to note that a comparison with a greater precision (i.e. without dropping higher derivative terms) will be possible if the effective action for LQC could be computed including higher time derivatives of the scale factor. However, in order to compare different underlying quantum theories, the homogeneous and isotropic sector alone cannot be sufficient since only certain combinations of the $c_{m,n}$'s can get constrained.

The proposed approach of comparing different quantum theories at the level of effective actions (really at the level of equations of motion since we do not worry about total derivative terms) is a preliminary one and can be quantitatively useful only when the effective actions at both the full and the reduced level are independently and reliably computable. In the

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[6] The combinations of these coefficients will in general be different in the two methods.

absence of availability of such actions, one can at best proceed with a qualitative comparison.

If effective actions are available for different classical sectors, then the demand that these be obtained from corresponding reductions from a *common* full theory effective action, would be restrictive. This provides a motivation for obtaining effective actions for several different classically reduced models [7].

Acknowledgments

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[7] While this work was being written up, the eprint of [12] appeared which has some overlap with our results. Our perspective and approach is however different. We focus on the effective action for the gravitational sector, discuss the Lovelock actions as a candidate and point out the appropriateness of $\bar{\mu}$-scheme. We do not discuss the matter sector which is addressed in the cited work.

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